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Formation & Solution of Differential Equation

There are two types of diff. eqn :-

① Ordinary Diff. Eqn :- The diff. eqn. which involve only one independent variable are called ordinary diff. eqn. For example —

(i) $\frac{dy}{dx} = \frac{1+y}{1+x}$ (ii) $\frac{dy}{dx} = \frac{x+y}{x-y}$ (iii) $y = x\left(\frac{dy}{dx}\right) + \left(\frac{dy}{dx}\right)^3$

are ordinary diff. eqns.

② Partial Diff. Eqn. :- If there are two or more independent variables in a diff. eqn., then its said to be partial. For example —

(i) $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = c$ (ii) $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 2z$ (iii) $\frac{\partial^2 y}{\partial t^2} = c^2\frac{\partial^2 y}{\partial x^2}$

are partial diff. eqns.

Order of Diff. eqn :- The order of a diff. eqn. is the order of the highest derivative (diff. coeff.) involved in its expression. For example

$\left(\frac{dy}{dx}\right)^2 + \left(\frac{dy}{dx}\right)^3 + 4x = 0$ is the diff. eqn. of the first order.

Here the maxm. derivative of y w.r.t. x is $\frac{dy}{dx}$

Similarly,

$\frac{d^2y}{dx^2} + 4y = e^x$ and $\left\{1+\left(\frac{dy}{dx}\right)\right\}^{3/2} = K\frac{d^2y}{dx^2}$

are of second order. [maxm. derivative of y w.r.t. x in $\frac{d^2y}{dx^2}$]

Degree of diff. eqn. can be defined as the degree of the highest order of diff. coefficient when the eqn. has been made rational [Free from any radicals] and integral as far as the diff. coeff. are connected concerned.

Thus $\left(\frac{dy}{dx}\right)^2 - 5\frac{dy}{dx} + 6y = 0$ [Though of First order] is of second degree. Now consider the diff. eqn.

$$\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{3/2} = K\frac{d^2y}{dx^2}$$

Squaring it (so that it may be rationalised), we get

$$\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^3 = K^2\left(\frac{d^2y}{dx^2}\right)^2.$$

Since $\frac{d^2y}{dx^2}$ occurs Squared, we find that the given diff. eqn. is of Second degree.

Similarly,

(i) $y = x\frac{dy}{dx} + \left(\frac{dy}{dx}\right)^3$, is of first order and third degree

(ii) $\left(\frac{dy}{dx}\right)^2 - 10\frac{dy}{dx} + 2y = 0$, is of first order and Second degree

(iii) $\frac{d^2y}{dx^2} - 7\frac{dy}{dx} + 10y = e^x$, is of second order and first degree

(iv) $\frac{d^3y}{dx^3} - 6\frac{d^2y}{dx^2} + 11\frac{dy}{dx} - 6y = 0$, is of third order and First degree

(v) $\left(\frac{d^3y}{dx^3}\right)^2 - 2\frac{d^2y}{dx^2} + y = 0$, is of third order and second degree

(vi) $\left(\frac{d^4y}{dx^4}\right)^3 + 4\frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} = 0$

is of Four order and third degree.

Here maxm derivative of y w.r.t. x is $\frac{d^4y}{dx^4}$ and has power as degree 3.